The Interview in Mathematics Education: The Case of Mental Computation

Ann Heirdsfield

Queensland University of Technology

a.heirdsfield@qut.edu.au

Use of clinical interview is becoming a significant aspect of many numeracy projects. It is important for teachers to identify children's understanding and misconceptions at all stages in the learning cycle, and the clinical interview appears to be an appropriate technique for gathering information on children's thinking. This paper explores the development of a conceptual framework used as a basis for an investigation into cognitive aspects associated with mental computation. Examples of tasks from clinical interviews which were based on this conceptual framework are described.

Numeracy initiatives around the nation are advocating teachers' assessing and recording information about students' mathematical progress. Assessment strategies should form part of ongoing planning. While there is a huge variety of assessment strategies, the interview is one method that appears to be a very effective method of obtaining information about children's mathematical understanding. The value of Piaget's revised clinical interview technique has been described by several researchers, for instance, Ginsburg, Kossan, Schwartz, and Swanson (1983), Labinowicz, 1985, and Hunting (1997). Merrifield and Pearn (1999) suggest that the clinical interview provides a very effective method of gathering information on children's mathematical thinking.

There are Australian examples of the employment of clinical interviews by both researchers and teachers. The *Count Me In Too* (CMIT, Bobis & Gould, 1999) project incorporates a clinical interview based assessment instrument that is designed to diagnose children's strengths ad weaknesses in arithmetical development. The *Early Numeracy Research Project* (ENRP, Clarke, 1999) has also developed an interview that project teachers administer to their students. The data form the basis for determining growth points for each child (Gervasoni, 2000). The interview tasks were developed from a framework, which owed much of its development to the *Count Me In Too* project. In Queensland Government schools, teachers are required to identify some Year 2 children who then participate in Validation Tasks in the form of interviews (Year 2 Diagnostic Net). One outcome is that the results of these interviews can inform teachers of students who are in need of intervention. An intervention program, *Mathematics Intervention* (Merrifield & Pearn, 1999) uses a clinical interview administered by specialist teachers to identify children at risk of not learning mathematics.

Teachers and researchers might be considered alike in that it would be expected that both groups aim to gain insight into how children learn and think, in order to improve teaching and learning. Several research projects, for instance, *Count Me In Too* (Stewart, Wright, & Gould, 1998) and *Cognitively Guided Instruction* (Carpenter, Fennema, Franke, Levi, & Empson, 1999) encourage teachers to base their instruction on knowledge of children's thinking. The researcher can afford the luxury of isolating a problem to investigate, building upon a conceptual framework, and often being able to withdraw children from the classroom to interview on a one-to-one basis, and. In contrast, the teacher often has to deal with more than one child at a time, and complex interactions impinging upon the classroom environment. While interview tasks are being developed for specific purposes (e.g., CMIT, ENRP), interviews can be developed by teachers to help them gain insight into children's knowledge and understanding. Large, funded projects run by academics base their clinical interviews on frameworks well situated in a knowledge framework and a theory of understanding. Teachers, also, should position clinical interviews (and teaching for that matter) in a framework based on a theory of understanding, sound content knowledge, and sound pedagogical content knowledge. Without these aspects, I believe that the interview might only reveal what a poorly prepared pen and paper test can reveal. In this paper, theories of understanding are discussed, a conceptual framework for my area of interest (mental computation) is presented, and finally, examples of tasks from clinical interviews aimed at investigating cognitive aspects of mental computation are described.

Understanding and knowledge

Hiebert and Carpenter (1992) suggested that understanding occurs when a fact, idea, or procedure is part of a network of interconnected facts, ideas, and procedures; and this network is connected to other networks in a meaningful way. They stated that a description of understanding encompasses structured internal representations, and connections both *within* internal representations and external representations, and *between* internal and external representations. In order to think about and manipulate mathematical ideas, they need to be represented internally, in a way that allows the mind to operate on them. Moreover, Hiebert and Carpenter (1992) proposed that understanding is generative in that new connections are constructed. As a consequence, understanding promotes remembering as connections are made between new and existing knowledge; and transfer is enhanced as similarities and differences are noted in the connections.

Putnam, Lampert, and Peterson (1990) categorised understanding from a cognitive perspective as having five themes.

1. *Understanding as representation.* The learner is able to move flexibly among and between external representations and internal cognitive representations.

2. *Understanding as knowledge structures.* Implicit knowledge is made explicit, involving rich, accessible schemata and domain specific knowledge.

3. *Understanding as connections among types of knowledge.* Distinctions are made between conceptual and procedural knowledge, and between formal and informal knowledge.

4. *Learning as active construction of knowledge.* Learners impose their existing frameworks to reorganise and integrate new knowledge.

5. *Understanding as situated cognition.* Rather than viewing cognition as existing within the mind of the individual, it is perceived as being situated in physical and social contexts.

Skemp (1989) spoke of *relational understanding* and *instrumental understanding*. Relational understanding requires the learner to build up interconnected, structured knowledge, whereas, instrumental understanding can be applied to very specific situations, and can be acquired through "habit learning" (Skemp, 1989, p. 32). Moreover, Carpenter (1986, p. 113) referred to "a rich network of relationships between pieces of information" as *conceptual knowledge,* and *procedural knowledge* as "step-by-step procedures". What was important was not the distinction between the two, rather, the links between the two, as both are essential.

Rittle-Johnson and Alibali (1999) also discussed relationships between conceptual and procedural knowledge. They suggested that children's conceptual understanding influenced the procedures they used. In a study of fourth and fifth grade children's conceptual understanding of equivalence (in mathematics) and their procedures for solving equivalence problems, conceptual instruction resulted in increased conceptual understanding and generation and transfer of a correct procedure. In contrast, although procedural instruction resulted in increased conceptual understanding, there was only limited transfer of the taught procedure. It was posited that conceptual knowledge might have a greater influence on procedural knowledge than the reverse.

As a result of investigation into expert mathematics instruction and the development of student knowledge, Leinhardt (1988) developed a theory of knowing which comprised four different types of knowledge, namely, *intuitive*, *concrete*, *computational*, and *conceptual*. *Intuitive knowledge* was not the result of direct teaching, and could be used to solve mathematical problems before instruction (e.g., Carpenter & Moser, 1984). Intuitive knowledge could also develop indirectly from instruction. *Concrete knowledge* was the knowledge gained by using real world, concrete (e.g., counters, MAB), or pictorial systems. Leinhardt (1988) described *computational knowledge* as procedural knowledge, not only in the sense of procedures for computation, but also application. Although this type of knowledge, Leinhardt suggested, constitutes the major part of the traditional school curriculum, she pointed out that success at computation did not guarantee understanding. According to Leinhardt, *principled conceptual knowledge* was "the underlying knowledge of mathematics from which the computational procedures and constraints can be deduced" (Leinhardt, 1988, p. 122). Further, she believed that, as competence grew, *intuitive* and *principled knowledge* converged to form a base "from which unique generative solutions can be formed and into which computation procedures can be nested and legitimized" (Leinhardt, 1988, p. 123). Finally, the four types of knowledge were not hierarchical, building on one another, but rather, the combination of the four and connections would represent understanding.

Perkins, Crismond, Simmons, and Unger (1995) posited that to be able to build and extend upon understanding, one needs *knowledge*, *representation*, *retrieval mechanisms*, and *construction mechanisms*. The *knowledge dimension* referred to content knowledge and strategies, for instance, problem solving strategies, higher-order thinking strategies, and metacognitive strategies. The *representation dimension* allowed students to connect new knowledge to old knowledge, and restructure old knowledge (complementary conceptual anchors). The *retrieval dimension* referred to retrieval of knowledge and representations from long-term memory. Two aspects of retrieval were mentioned - "pop up" and "dig out". "Pop up" retrieval occurred when links were made to related information structures, "which simply come to mind" (Perkins, Crismond, Simmons, & Unger, 1995. p. 79). In contrast, "dig out" retrieval generally required metacognition. The *construction dimension* referred to the mechanisms of learning. Two aspects of learning were discussed - "catching on" and "working through". "Working through" required metacognitive strategies, for instance, reflection and testing ideas. "Catching on" learning would be evidenced in performance-oriented students, and "working through" would be evidenced in mastery-oriented students. Overall, access was considered a complex process, far more complex than simple rote learning.

Alexander and Judy (1988) also argued that expertise involved not only an extensive knowledge base, but an accessible one as well. They added that *strategic knowledge* (knowledge of domain specific and across domain strategies, and metacognitive strategies) was essential for activating the *content knowledge*, and facilitating learning. Also, *domain-specific knowledge* was necessary for utilisation of *strategic knowledge*. *Domain-specific knowledge* was defined as the declarative (knowing what), procedural knowledge (knowing how, including strategic knowledge), and conditional knowledge (when and where to access), possessed by the individual relative to a particular field of study. They reported that students who were identified as possessing good conceptual knowledge exhibited reflective thought. Thus, experts possessed extensive *domain-specific knowledge* that permitted them to establish a context and categorise problems on the basis of underlying principles or concepts. Further, they possessed *strategic knowledge* that enabled them to apply and monitor the *content knowledge*. In other words, experts perceived deep structure.

Alexander and Judy (1988) also posited that acquisition and utilisation of both *domain* and *strategic knowledge* could be affected by motivational and social-contextual factors. In other words, it was insufficient to have access to rich information or strategies, if motivational or affective factors were negative.

It has been argued elsewhere (Brown & Palincsar, 1989) that an aspect of a study of "knowing" should address Vygotsky's zone of proximal development (Vygotsky, 1978). Vygotsky argued that a child's level of development cannot be understood unless both the child's actual developmental level (determined by independent activity) and potential developmental level (determined by guidance provided to the child) were established. The zone of proximal development is the "distance between the actual developmental level ... and the level of potential development" (Vygotsky, 1978, p. 86). Children at the same actual level of development may have different zones of proximal development.

Understanding and knowing in mental computation

Anghileri (1999) claimed that mental computation was calculating *with* the head, rather than merely, *in* the head, that is, mental computation is calculating using strategies with understanding. Thus, proficiency in mental computation was not confined to accuracy, but also included flexibility of strategy choice. Therefore, the factors that influence mental computation consist of those that affect flexibility as well as accuracy.

Here, understanding is viewed as interconnected networks of knowledge representations and structures, where access is readily available. Therefore, in order to establish a framework for mental computation, literature that reports factors associated with mental computation was consulted. The literature has shown that mental computation may be viewed as a subset of number sense, as students who exhibit proficiency in mental computation also display number sense (e.g., McIntosh, 1996; McIntosh, Reys, & Reys, 1992; Sowder, 1990, 1992). Research on mental computation has proposed specific connections among mental computation and aspects of number sense, in particular, number facts knowledge and estimation (e.g., Heirdsfield, 1996; Sowder, 1992). Other research relating to computation (in particular, children's natural strategies) has reported connections with number and operation (the effects of operation on number) and numeration, for example, place value, (e.g., Kamii, Lewis, & Jones, 1991).

Sowder and Wheeler (1989) stated that the abilities to compute mentally and estimate proficiently were related skills. Research (Reys, Bestgen, Rybolt, & Wyatt, 1980; Reys, 1986; Reys, 1991) indicated that good estimators possessed a variety of skills and were flexible in the way they think about numbers. However, poor estimators had little understanding of what estimation meant. They usually tried to calculate exact answers, and then give an estimate from that answer. They also applied rigid algorithms that had been taught in the classroom, with little understanding of the appropriateness of the strategy (Reys, 1991). Poor estimators who lacked number understanding often used rounding as the sole strategy at their disposal. Often, rounding to numbers that were more compatible with the computation involved was more appropriate and showed a higher level of number sense.

Plunkett (1979) and Sowder (1988) suggested that knowledge of basic number facts is a prerequisite for mental computation. If number facts are easily retrieved from long-term memory, working memory is available for more efficiently solving more complex mental problems, rather than having to attend to calculating what would normally be retrieved from memory (Resnick & Ford, 1981).

It would appear evident that proficient addition and subtraction mental computation should require expertise in numeration. Bednarz and Janvier (1988) considered place value as a prerequisite to learning operations. However, other research has suggested that prerequisite numeration knowledge may not be as important to addition and subtraction as was thought. Usnick and Engelhardt (1988) found that knowledge of numeration might not be a necessary prerequisite for learning multidigit operations. Rather, they posited that numeration might be improved in the context of learning operations like addition. Ross (1990) also supported the notion that teaching place value "separately as a prerequisite to double-digit addition and subtraction is ineffective and unnecessary" (p. 15). Further evidence for not teaching place value separately has come from Kamii and Joseph (1988), and Cobb and Merkel (1989) who investigated children's non-traditional strategies for operations and numeration concepts when neither traditional algorithms nor place value were taught. It was found that children could invent their own efficient algorithms without specific instruction in place value. Yet, their non-traditional strategies exhibited an understanding of place value.

Some research more specific to mental computation has indicated a link between place value understanding and mental computation (e.g., Kamii, Lewis, & Jones, 1991; Reys, 1985; Sowder, 1992). The acceptance of noncanonical representations may be important for mental computation. Resnick and Omanson (1987) reported that one child who was a proficient mental computer showed least inclination for canonical forms, preferring to explore different number representations. Other research (e.g., Fuson, Fraivillig, & Burghardt, 1992; Miura, Okamoto, Kim, Steere, & Fayol, 1993) has focused on children's problems with understanding place value; for instance, misconceptions of number words, viewing numbers as one-to-one collections (42 is 42 units), canonical base 10 representations (42 is 4 tens and 2 ones), and noncanonical base 10 representations (42 is 3 tens and 12 ones).

An understanding of the effects of operation on number appears to be essential for flexible mental computation, as some of the strategies that good mental computers employ include decomposing and recomposing number to best suit the operations (Sowder, 1988). Students may not have a formal knowledge of these properties, but possess a working knowledge, for instance, knowing that 126+99 can be solved by (126+100)-1. This strategy would be both efficient and reduce demands on working memory, compared with a pen and paper algorithm performed mentally. Hope and Sherrill (1987) found that skilled mental computers when compared with unskilled ones used a variety of such strategies.

In Figure 1, the conceptual framework is illustrated and examples of the connections are presented. It is recognised that understanding is influenced by more than just cognitive factors. However, as the purpose in this paper is to describe interviews, so as not to make the task too complex, the discussion will be restricted to cognitive factors.

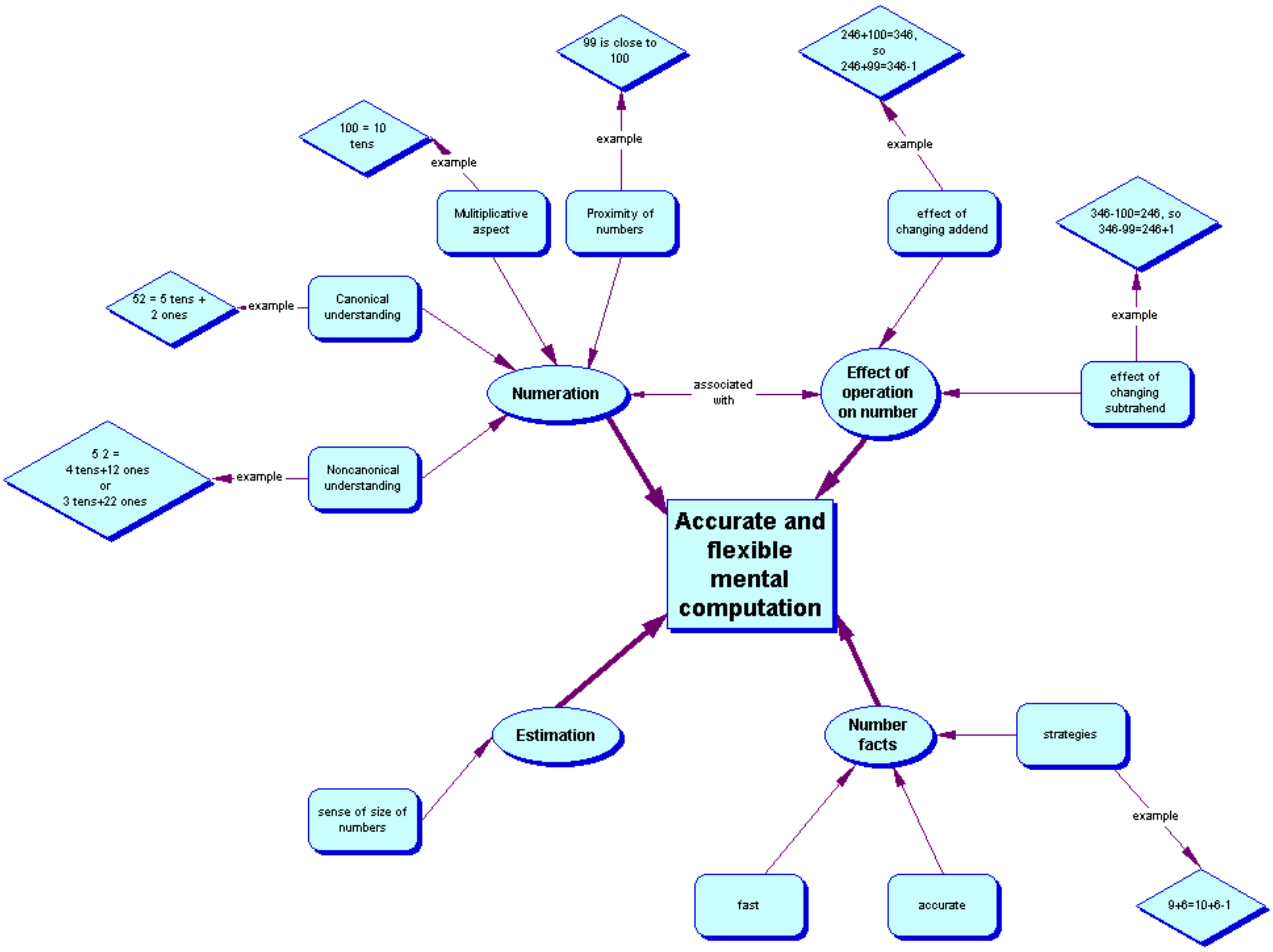


Figure 1. Conceptual framework for accurate and flexible mental computation (cognitive aspects)

Once a framework is established, interview tasks can be developed. In the next section the value of interviews in investigating each of the factors is discussed, and some possible tasks are presented.

Interview tasks

Mental computation

Consideration of current mathematics curriculum documents should inform the selection of number combinations to be presented. Also, tasks at a lower level and a higher level should be included. To encourage the use of mental strategies, number combinations that lend themselves to computing mentally should be selected. Further, the presentation format should be a consideration. In Figure 2, a suggested format is presented. The picture (and numerals) is presented on card and the question is spoken.

"What is the total cost of the two computer games?"

Figure 2. Example of mental computation task

As accuracy is not the only factor involved in mental computation, identification of the strategy employed can be established by asking the child to explain the solution strategy after calculating. Some researchers suggest asking the children to talk aloud while calculating, however, I have found that this interferes with young children's thought processes. So students were directed to explain their solution strategies, after they had completed their calculations. A wide variety of mental addition and subtraction strategies has been identified in the literature (e.g., Beishuizen, 1993; Cooper, Heirdsfield, & Irons, 1996; Kamii, 1989; Reys, Reys, Nohda, & Emori, 1995; Thompson & Smith, 1999). While it is useful to use a categorisation scheme, teachers will find that there will be strategies children use that cannot be easily identified. What is important about identifying the strategy is that the appropriateness of the strategy can be judged, and we might find out what the child is thinking, how they're doing things.

Children's discussions are useful for not only discovering their understandings, but also any misconceptions, for instance, a child's calculations for the following examples appeared to indicate misconceptions relating to numeration (regrouping). However, when the child was asked to explain his strategy, his responses were:

7 5	5 take 8, can't do, so regroup. 15 take 8 is 7. 6 take 2 is 5.
- 2 8	
5 7	
6 8	8 take 9, can't do, so regroup. 18 take 9 is 9. 5 take 2 is 4.
- 2 9	
4 9	

In both cases, number fact errors caused the miscalculations. Why he made errors with easy number facts and not the more difficult ones was not established.

We might also want to establish if the child has the potential to access efficient strategies. Van der Heijden (1994) used a Vygotskian approach to investigate mental addition and subtraction of primary school children. Vygotsky's "zone of proximal development" was considered an important aspect of qualitative assessment of children's mental addition and subtraction proficiency, defined by speed, accuracy and efficient strategy use. Pre-determined scaffolding questions were presented to children who did not employ what was considered efficient mental procedures. Examples of scaffolding questions that might be used are:

1. Can you think of another way of solving the problem?
2. What is (e.g., 99) close to?
3. Can you work with this number?
4. What can you do now?

If the child successfully employs a more efficient strategy after scaffolding, it might be interesting to ask him/her which strategy is preferred. Van der Heijden (1994) reported that students possessed a considerable potential for efficient strategies, yet they solved the problems originally by using efficient mental strategies in only thirty percent of the cases. The students generally agreed that the efficient strategy was easier. However, I have found that many children will state that their original, less efficient strategy is the preferred strategy, because they "are used to it".

A strength of interviewing children for mental computation is that an awareness of the variety of strategies that children possess can be developed. Further, as results of formal testing cannot analyse a student's understanding and can often lead to incorrect assumptions about a student's ability or the strategy used, the interview technique is preferred. In this study, this method of data collection can result in the gathering of rich data that can be analysed in terms of appropriate strategy use, rather than only accuracy.

Computational estimation

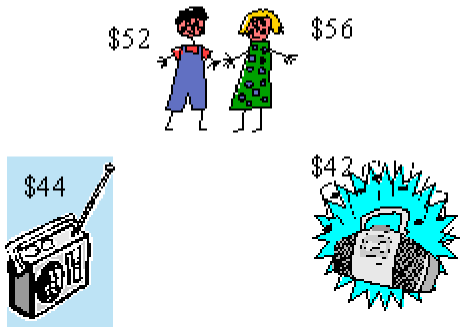
A factor in proficient mental computation is computational estimation. Research on computational estimation has presented several difficulties. It cannot be assessed by standardised pen and paper tests. Discussions with the subjects hold the key to the methods and understanding of the individual's skill in estimation. That is, "skill" in computational estimation does not depend on "getting the right answer", as context and appropriateness of strategy are also involved in proficiency. In a study of estimation performance of middle school students (Threadgill-Sowder, 1984), interviews revealed a number of correct answers with incorrect explanations. Thus, interviews can give better results than would be discovered from written tests. At best, tests might identify students who are good estimators (i.e., score high on tests), and might identify related skills (Rubenstein, 1985); they cannot identify strategies or thinking processes.

Schoen, Blume, and Hart (1987) tested estimation processes used by students in Grades 5 through 8, by open ended testing, multiple choice testing and interviews. Strategies were identified in the open-ended testing by analysing answers and determining the estimation process most likely to be used. However, it was difficult to distinguish some strategies from answers that were obtained, thus strategy identification could only be surmised. Multiple-choice responses were compared with interview responses, in order to validate possible strategies used. Analysis of interviews resulted in obvious identification of the strategies used. This could hardly be said for the analysis of test results.

There is also possibility that students do not estimate, that is, they compute mentally (Levine, 1982; Reys, Bestgen, Rybolt, & Wyatt, 1982), or compute mentally and then round to produce an estimate (Sowder & Wheeler, 1989). By presenting multiple choice format, Schoen, Blume, and Hoover (1990), endeavoured to force students to use particular estimation strategies, and not calculate. A small sample of students (not included for testing) was interviewed on ten items from the paper-pencil tests. However, as the investigators admitted, the multiple choice form of the tests forced students into particular estimation strategies, and did not permit students to choose an estimation strategy they may have felt to be appropriate.

Reys, Bestgen, Rybolt, and Wyatt (1982) also formulated an estimation test, in an open-ended format, presented visually, and included both straight computation items and application items. The test results were used for selection of 59 good estimators to participate in interviews to identify strategies employed by good estimators. The interviews were useful for identifying, not only strategies, but also characteristics of good estimators. Again, this could not have been achieved by testing alone.

It would appear that strategies are not only important for mental computation, but also computational estimation, therefore, tasks should be presented to children in an interview situation. Some examples of estimation tasks, based on Case and Sowder (1990), Heirdsfield (2001), Rubenstein (1985), and Threadgill-Sowder (1984) are presented here (Figure 3). Again, curriculum documents should be consulted for degree of difficulty. What is obvious about the following examples is that the estimation strategy, *rounding*, is not an appropriate strategy for either example.



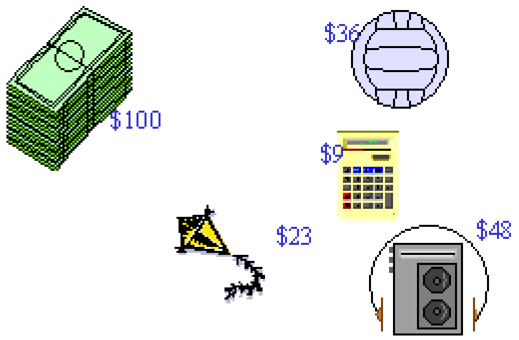
You and a friend go shopping for cassette players.

Your friend has $52 and buys a player for $44.

You have $56 and buy a player for $42.

Who has more money left?

Why?



Is $100 enough?

How can you tell?

Figure 3. Examples of computational estimation tasks

Number facts knowledge

Another factor in proficient mental computation is number fact knowledge. Components of number fact knowledge that are important are speed, accuracy and efficient number fact strategies (when the number fact is not known by recall). Accurate and speedy recall of basic number facts is a major objective of primary school mathematics teaching (Baroody, 1985; Thornton, 1990). However, children do not easily learn number facts as basic recall. Children will originally count (*count all*, *count on*, etc.). Then, they might develop derived facts strategies (e.g., *near doubles*, *through 10*). Research on acquisition of number facts (Cobb, 1983; Gray, 1991) indicated that learning derived facts strategies were aids to mastering number facts. It would also appear evident that some derived facts strategies might be extended to mental computation strategies, for instance, the same *through 10* strategy used to solve 6+9 could be used to solve 65+99.

Timed written tests do not indicate what strategy a child uses. Some children are very fast counters! If we want to know whether they know their facts by recall, whether they are using derived facts strategies, or whether they are still counting, we need to ask them. The procedure I have followed (Heirdsfield, 1996, 2001) is to set eight addition and eight subtraction number facts. The test was designed to allow such strategy use (Thornton, 1990) as:

- *near doubles*: 7+8, 6+8, 16-7, 15-8,
- *build to 10*: 8+5, 7+4, 12-4,
- *pattern with 9*: 9+6, 3+9, 17-9,
- *use another fact*: 8+5, 6+8, 9-3, and
- *use addition (for subtraction)*: 11-8.

It was not expected that students necessarily would use these particular strategies for the specific examples, that is, a *near doubles* example need not be solved using a *near doubles* strategy (e.g., 9+8 could be solved using *near doubles*, *pattern with 9*, and *also build to 10*). However, it was essential to include questions that would permit use of these strategies, if the student has access to them. The children were directed to write the answers for one set (addition or subtraction) against the questions. At the end of each set, the children were asked to explain how they arrived at each answer. Students were also asked if they could think of another solution strategy (particularly if they had counted). Obviously, if they knew the number fact, they were not asked to solve the question in another way.

Numeration

To be able to manipulate numbers mentally requires an understanding of partitioning of number (e.g., 34 is not only 3 tens and 4 ones in canonical form, but also 2 tens and 14 ones in noncanonical form) and manipulating numbers (e.g., 34 + 21 is 44, 54, 55) (Resnick, 1983). Further, it seems apparent that children need to conceptualise numbers as entities, rather than symbols side by side (e.g., 42 is made up of 4 and 2, rather than 4 tens and 2 ones). That is, they need to comprehend numbers more in terms of the multiplicative nature of our number system than as hundreds, tens, and ones place value (Bednarz & Janvier, 1982).

Therefore, to investigate numeration, tasks addressing the understanding of canonical and noncanonical representations of number, and understanding of the multiplicative structure of the number system might be included. Some suggested tasks based on those of Bednarz and Janvier (1982), Resnick and Omanson (1987), Ross (1990), and Sierink and Watson (1991) are presented below (Figure 4). These tasks were designed to be presented to eight-year old children (Year 3). Recently, these tasks have been successfully adapted for children in Years 2 and 4, by decreasing/increasing the number of digits in tasks 2 to 5.

Task 1 Digit correspondence (Kamii, 1986).

Present 16 counters. "Please count these for me". "Write the number."

Circle 6. "Does this have anything to do with the number of counters you counted?" / "Show me the counters which refer to this number."

Circle 1. "Does this have anything to do with the number of counters you counted?" / "Show me the counters that refer to this number." At this stage, one child picked up one counter, rather than 10.

Task 2 Non canonical understanding (Ross, 1990).

Use MAB (no more than 22 ones). Make 52. Make it another way, and another way.

Task 3 (Resnick & Omanson, 1987)

32 and 73. Which "3" is worth more? Why?

Task 4 Writing and reading two- and three-digit numbers and canonical and non-canonical understanding (Sierink & Watson, 1991)

Read and write 2 and 3 digit numbers.

"Please write: 15, 54, 103, 690".

"Please read these numbers: 19, 83, 209, 560" (presented on separate cards).

What can you tell me about these numbers (underlined)? The children were encouraged to describe the numbers in terms of canonical and noncanonical forms (similar to task 2). If they experienced difficulty, they were permitted to use MAB.

Task 5 Ordering multidigit numbers by interchanging digits (Bednarz & Janvier, 1982)

Given a set of 4 digit cards (8, 5, 3, 0), make the largest number possible, and then the smallest number possible. The children were asked to explain how they knew the number was largest/smallest.

If this is too difficult, replace 0 with 2.

If it is still too difficult, use 3 digits (8, 5, 3), then (8, 5, 0).

Task 6 The children were directed to calculate the first example, and then write the answers to the remainder, without calculating. They were asked to explain how they knew the answers.

7 + 6

70 + 60

700 + 600

7	70	700
+ 6	+ 60	+ 600

Figure 4. Numeration tasks

Effect of operation on number

Another factor in the successful employment of efficient mental computation strategies (for addition and subtraction) is understanding the effect of operation on number, in particular, understanding the effect of changing the addend and the subtrahend. To investigate understanding of the effects of operations on number, examples are drawn from McIntosh, Reys, and Reys (1992), Sowder (1992), and Sowder and Wheeler (1987) (Figure 5). The first number sentence is given to the children. They are asked to complete the other examples by using the original number sentence or one they have already solved (without actually calculating). Understanding the effect of changing the addend and subtrahend seems to be particularly pertinent for the employment of the *wholistic* strategies in both addition and subtraction (e.g., 246+99=246+100-1).

43 + 26 = 69

43 + 27 =	430 + 260 =
42 + 27 =	260 + 430 =
69 - 26 =	69 - 43 =
70 - 26 =	70 - 43 =
70 - 25 =	70 - 44 =
70 - 27 =	690 - 260 =
146 + 100 = 146 + 99 =	257 - 100 = 257 - 99 =

Figure 5. Effect of operation on number tasks

Conclusion

It is proposed here that in order to gather information about a child's mathematical learning, interviews play an important role. By listening to children's explanations, conceptual understanding and misconceptions can be clarified. Tasks need to be situated in a conceptual framework, based on a theory of understanding. Therefore, it would appear that teachers need sound pedagogical content knowledge. However, Hunting (1997) suggests that this need not be so.

> Clinical approaches to assessment can open the door for teachers to begin to expand their experience of how children's minds work mathematically... Clinical interviews allow students to teach teachers. (p. 160)

From anecdotal evidence, this appears to be substantiated. When the Year 2 Net was instigated in Queensland, discussions with teachers indicated that some viewed the interviews that they had to conduct with their students as very informative of individual student's understanding. Prior to interviewing students, some of the teachers had not envisaged questioning and listening as important parts of assessment or instruction. The validation tasks (Year 2 Validation Tasks) are determined for the teachers by specialists, and therefore, the teachers did not need to have a sound knowledge of the concepts being assessed. However I would suggest that by listening to the children, the teachers were learning something about how the children think, and how concepts are connected. If teachers are to base instructional practices upon children's knowledge (or misconceptions), it would appear that by employing an interview, the teacher can attempt to understand children's understanding.

References

Alexander, P. A., & Judy, J. E. (1988). The interaction of domain-specific and strategic knowledge in academic performance. *Review of Educational Research, 58*(4), 375-404.

Anghileri, J. (1999). Issues in teaching multiplication and division. In I. Thompson (Ed.), *Issues in teaching numeracy in primary school* (pp. 184-194). Buckingham: Open University Press.

Baroody, A. (1985). Mastery of basic number combinations: Internalization of relationships or facts? *Journal for Research in Mathematics Education, 16*(2), 83-98.

Bednarz, N., & Janvier, B. (1982). The understanding of numeration in primary school. *Educational studies in mathematics, 13*(1), 33-57.

Bednarz, N., & Janvier, B. (1988). A constructivist approach to numeration in primary school: Results of a three year intervention with the same group of children. *Educational studies in mathematics, 19,* 299-331.

Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education, 24*(4), 294-323.

Bobis, J., & Gould, P. (1999). The mathematical achievement of children in the Count Me In Too program. In J. M. Truran & K. M. Truran (Eds.), *Making the difference* (pp. 84-90). Sydney, NSW: MERGA.

Brown, A. L., & Palincsar, A. S. (1989). Guided, cooperative learning and individul knowledge acquisition. In L. B. Resnick (Ed.), *Knowing, learning and instruction. Essays in honor of Robert Glaser* (pp. 393-451). Hillsdale, NJ: Erlbaum.

Carpenter, T. P. (1986). Conceptual knowledge as a foundation for procedural knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 113-132). Hillsdale, NJ: Lawrence Erlbaum Associates.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics. Cognitively guided instruction*. Portsmouth, NH: Heinemann

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15*(3), 179-202.

Case, R., & Sowder, J. (1990). The development of computational estimation: A neo-Piagetian analysis. *Cognition and Instruction, 7*(2), 79-104.

Clarke, D. M. (1999). Linking assessment and teaching: Building on what children know and can do. In Early Years of Schooling Branch (Eds.), *Targeting excellence: Continuing the journey* (pp. 8-12). Melbourne; Author.

Cobb, P. (1983). *Children's construction of thinking strategies to find sums and differences*. Paper presented at the American Educational Research Association, Montreal.

Cobb, P., & Merkel, G. (1989). Thinking strategies: Teaching arithmetic through problem solving. In P. Trafton & A. Schulte (Eds.), *New directions for elementary school mathematics. 1989 yearbook* (pp. 70-81). Reston: National Council of Teachers of Mathematics.

Cooper, T. J., Heirdsfield, A., & Irons, C. J (1996). Children's mental strategies for addition and subtraction word problems. In J. Mulligan & M. Mitchelmore (Eds.), *Children's number learning* (pp. 147-162). Adelaide: Australian Association of Mathematics Teachers, Inc.

Fuson, K., Fraivillig, J., & Burghardt, B. (1992). Relationships children construct among English words, multidigit base-ten blocks, and written multidigit addition. In J. J. D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 39-112). Netherlands: Elsevier Science Publishers.

Gervasoni, A. (1999). Using growth point profiles to identify year 1 students who are at risk of not learning school mathematics successfully. In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (pp. 275-283). Perth, WA: MERGA.

Ginsburg, H., Kossan, N., Schwartz, R., & Swanson, D. (1983). Protocol methods in research on mathematical thinking. In H. P. Ginsburg (Ed.), *The development of mathematical thinking.* New York: Academic Press.

Gray, E. M. (1991). An analysis of diverging approaches to simple arithmetic: Preference and its consequences. *Educational Studies in Mathematics, 22*, 551-574.

Heirdsfield, A. M. (1996). *Mental computation, computational estimation, and number fact knowledge for addition and subtraction in year 4 children*. Unpublished master's thesis, Queensland University of Technology, Brisbane.

Heirdsfield, A. M. (2001). *Mental computation: The identification of associated cognitive, metacognitive, and affective factors*. Unpublished doctoral thesis, Queensland University of Technology, Brisbane.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-126). New York: Macmillan.

Hope, J. A., & Sherrill, J. M. (1987). Characteristics of unskilled and skilled mental calculators. *Journal for Research in Mathematics Education, 18*(2), 98-111.

Hunting, R. P. (1997). Clinical interview methods in mathematics education research and practice. *Journal of Mathematical Behavior, 16*(2), 145-165.

Kamii, C. (1989). *Young children continue to reinvent arithmetic - 2nd grade: Implications of Piaget's theory*. New York: Teachers College Press.

Kamii, C., & Joseph, L. (1988). Teaching place value and double-column addition. *Arithmetic Teacher, February*, 48-52.

Kamii, C., Lewis, B., & Jones, S. (1991). Reform in primary education: A constructivist view. *Educational Horizons. 70*(1), 19-26.

Labinowicz, E. (1985). *Learning from children*. Menlo Park, CA: Addison Wesley.

Leinhardt, G. (1988). Getting to know: Tracing students' mathematical knowledge from intuition to competence. *Educational Psychologist, 23*(2), 119-144.

Levine, D. R. (1982). Strategy use and estimation ability of college students. *Journal for Research in Mathematics Education, 13*, 350-359.

McIntosh, A. (1996). Mental computation and number sense of Western Australian students. In J. Mulligan & M. Mitchelmore (Eds.), *Children's number learning* (pp. 259-276). Adelaide: Australian Association of Mathematics Teachers, Inc.

McIntosh, A., Reys, B., & Reys, R. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics, 12,* 2-8.

Merrifield, M., & Pearn, C. (1999). Mathematics intervention. In Early Years of Schooling Branch (Eds.), *Targeting excellence: Continuing the journey* (pp. 62-70). Melbourne; Author.

Miura, I., Okamoto, Y., Kim, C., Steere, M., & Fayol, M. (1993). First graders' cognitive representation of number and understanding of place value: Cross-cultural comparisons - France, Japan, Korea, Sweden, and the United States. *Journal of Educational Psychology, 85*(1), 24-30.

Perkins, D. N., Crismond, D., Simmons, R., & Unger, C. (1995). Inside understanding. In D. N. Perkins, J. L. Schwartz, M. M. West, & M. S. Wiske (Eds.), *Software goes to school* (pp. 70-87). New York: Oxford University Press.

Plunkett, S. (1979). Decomposition and all that rot. *Mathematics in Schools*, *8*(3), 2-5.

Putnam, R. T., Lampert, M., & Peterson, P. L. (1990). Alternative perspectives on knowing mathematics in elementary schools. *Review of Research in Education, 16,* 57-150.

Resnick, L. B. (1983). A developmental theory of number understandings. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 109-151). Orlando, Florida: Academic.

Resnick, L. B., & Ford, W. W. (1981). *The psychology of mathematics for instruction*. Hillsdale, New Jersey: Laurence Erlbaum Association, Publishing.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology, vol. 3* (pp. 41-95). Hillsdale, New Jersey: Lawrence Erlbaum.

Reys, B. J. (1985). Mental computation. *Arithmetic Teacher, 32*(6), 43-46.

Reys, B. J. (1986). Estimation and mental computation: It's about time. *Arithmetic Teacher, 34*(1), 22-23.

Reys, R. E. (1991). *Research on computational estimation - What it tells us and some questions that need to be addressed*. Paper presented at the Gwinganna Computational Alternatives Conference, Gold Coast, Australia. August 2-9.

Reys, R. E., Bestgen, B. J., Rybolt, J. F., & Wyatt, J. W. (1982). Processes used by good computational estimators. *Journal for Research in Mathematics Education, 13*(3), 183-

201.

Reys, R. E., Reys, B. J., Nohda, N., & Emori, H. (1995). Mental computation performance and strategy use of Japanese students in grades 2, 4, 6, and 8. *Journal for Research in Mathematics Education, 26*(4), 304-326.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology, 91*(1), 175-189.

Ross, S. H. (1990). Children's acquisition of place-value numeration concepts: The roles of cognitive development and instruction. *Focus on Learning Problems in Mathematics, 12*(1), 1-17.

Rubenstein, R. N. (1985). Computational estimation and related mathematical skills. *Journal for Research in Mathematics Education, 16*(2), 106-119.

Schoen, H., Blume, G., & Hart, E. (1987). Measuring computational estimation processes. Paper presented at the annual meeting of the American Educational Research Association. Washington.

Schoen, H., Blume, G., & Hoover, H. (1990). Outcomes and processes on estimation test items in different formats. *Journal for Research in Mathematics Education, 21*(1), 61-73.

Sierink, T., & Watson, J. (1991). Children's understanding of place value. *Australian Journal of Early Childhood, 16*(4), 33-42.

Skemp, R. R. (1989). *Mathematics in the primary school.* London: Routledge.

Sowder, J. (1988). Mental computation and number comparisons: Their roles in the development of number sense and computational estimation. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 182-197). Hillsdale: NJ: Lawrence Erlbaum Associates.

Sowder, J. (1990). Mental computation and number sense. *Arithmetic Teacher, 37*(7), 18-20.

Sowder, J. (1992). Making sense of numbers in school mathematics. In G. Leinhardt, R. Putman & R. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 1-51). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Sowder, J., & Wheeler, M. (1989). The development of concepts and strategies used in computational estimation. *Journal for Research in Mathematics Education, 20*, 130-46.

Stewart, R., Wright, R., & Gould, P. (1998). Kindergarten students' progress in the count me in too project. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times* (pp. 556-563). Brisbane: MERGA.

Thompson, I., & Smith, F. (1999). *Mental calculation strategies for the addition and subtraction of 2-digit numbers.* Final report. University of Newcastle, Newcastle upon Tyne.

Thornton, C. (1990). Solution strategies: Subtraction number facts. *Educational Studies* *in Mathematics, 21*, 241-263.

Threadgill-Sowder, J. (1984). Computational estimation procedures of school children. *Journal of Educational Research, 77*(6), 332-336.

Usnick, V., & Engelhardt, J. (1988). Basic facts, numeration concepts and the learning of the standard multidigit addition algorithm. *Focus on Learning Problems in Mathematics, 10*(2), 1-14.

Van der Heijden, M. K. (1994). A Vygotskian approach on mental addition and subtraction up to hundred; Handy arithmetic for primary school children. In J. Van Luit (Ed.), *Research on learning and instruction of mathematics in kindergarten and primary school* (pp. 108-124). The Netherlands: Graviant Publishing Company.

Vygotsky, L. S. (1978). *Mind in society - The development of higher psychological processes.* Cambridge, MA: Harvard University Press.